

# Training III

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## Force Generation and Transmission

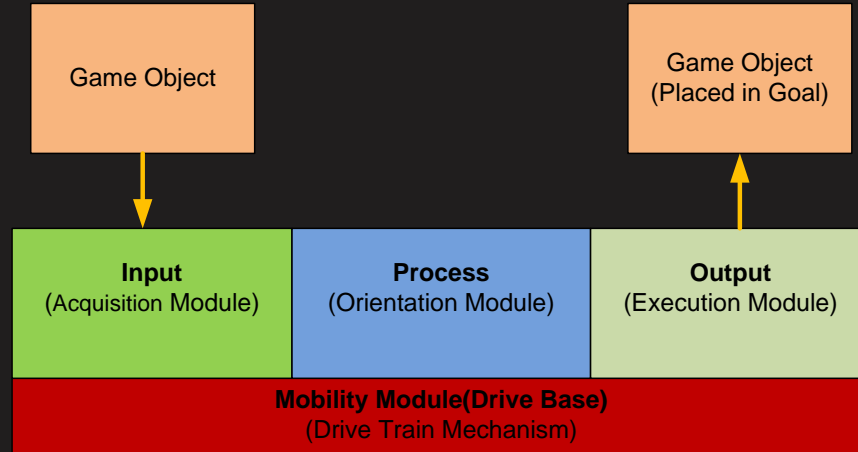
# Force Generation and Transmission Objectives

- Understand Energy Conversion to do Work on Robots
- Understand mechanical advantage through gears and pistons
- Understand mechanism energy calculations
- Understand motor electrical characteristics
- Understand motor / gear selection

# What is a FIRST Robot

A FIRST Robot is an programmable electro-mechanical machine that performs tasks through end effectors. It performs these tasks in either an autonomous or semi-autonomous mode.

FIRST Robot Block Diagram



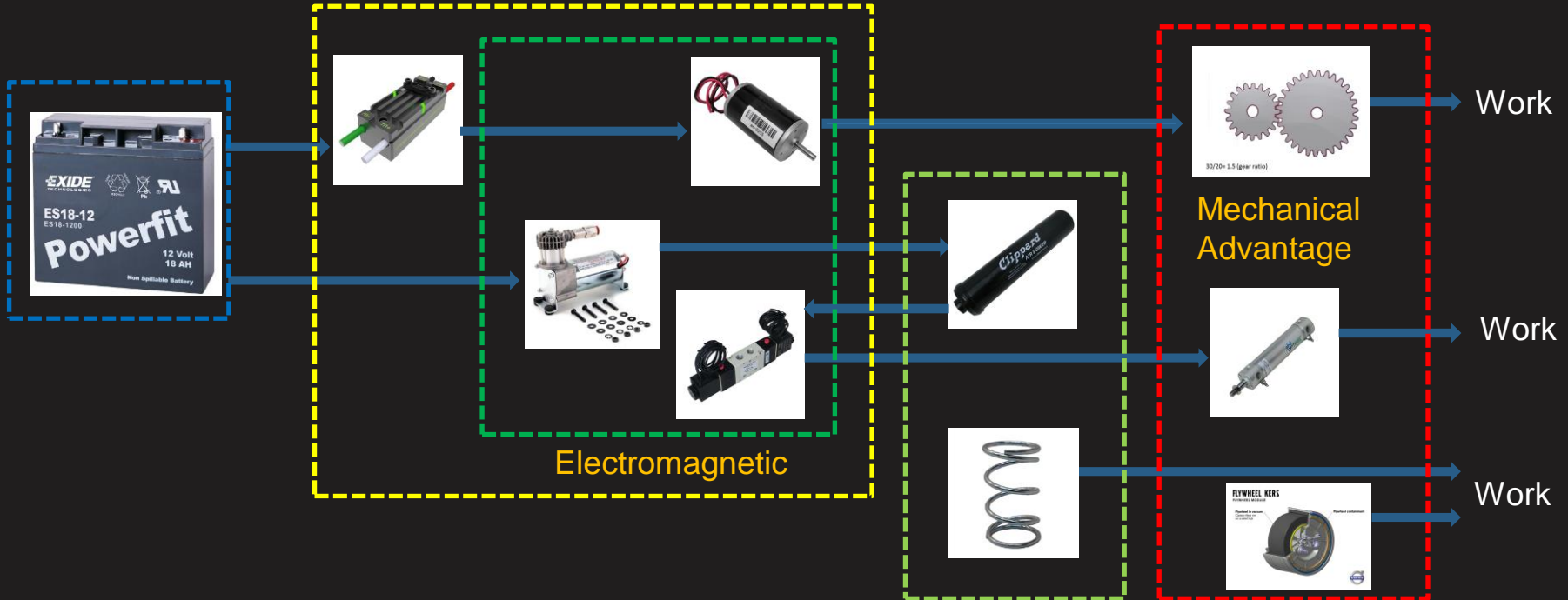
# Energy Conversion to Do Work

Chemical Energy

Electrical Energy

Mechanical Potential Energy

Mechanical Kinetic Energy



# Mechanical Energy (Linear)

Mechanical Energy is the energy acquired by objects upon which work is done

Mechanical Energy has two forms: Potential Energy and Kinetic Energy

$$E_{\text{mech}} = PE + KE;$$



Kinetic Energy(KE) – Energy of motion

$$KE = \frac{1}{2} \times m \times v^2;$$

Potential Energy(PE) – Stored energy of position

Gravitational:

$$PE = m \times g \times h$$

Elastic – energy stored in elastic materials – Spring, surgical tubing:

$$PE = \frac{1}{2} \times K \times x^2; \text{ where } K \text{ – spring constant}$$

Also - Compressed Air(Elastic):

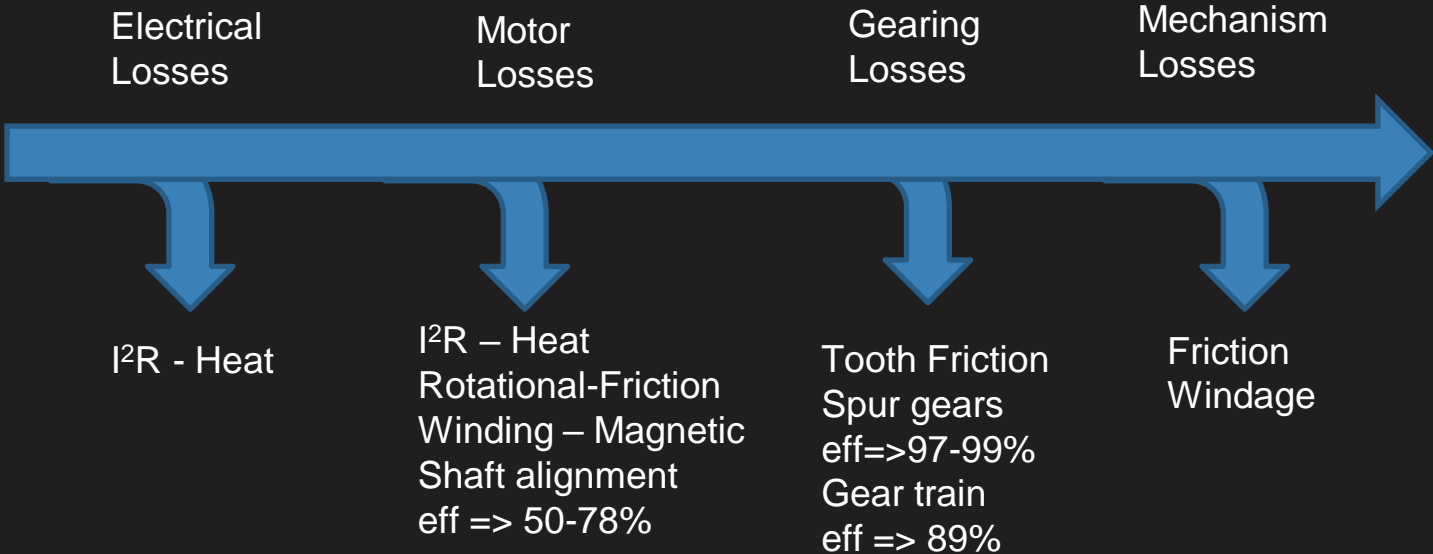
$$PE = P_i \times V_i \times \ln(P_f / P_i)$$

# Conservation of Energy

Conservation of energy:

“Energy can neither be created or destroyed; rather, it transforms from one form to another”  
Wikipedia

Energy OUT NOT EQUAL to Energy IN – Why?



# Energy ↔ Work

“Work is the Energy transferred by a Force”



“Energy is the ability to do work”

Wikipedia

Joule is a unit of energy and work

# Work

Work shifts energy from one system to another

$$\text{Work(Joules)} = \text{Force(Newtons)} \times \text{distance(meters)}$$

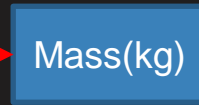
$$\text{Note: } J = Nm = (\text{kg} \times \text{m} / \text{s}^2) \times \text{m} = \text{kg} \times \text{m}^2 / \text{s}^2$$

$$\text{Linear Work: } W = F \times d$$

Force(Newtons)



Mass(kg)

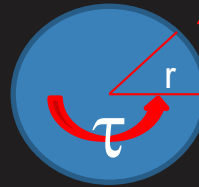


distance(meters)



$$\text{Rotational Work: } W = F \times r \times S$$

Force(Newtons)



S(radians)

$$\text{Torque(Twisting Force): } \tau = F \times r$$

# Power

Power is the rate at which work is done

$$\text{Power(watts)} = \text{Work(Nm)} / \text{Time(seconds)}$$

$$P = F \times d / s - \text{Note: 1 watt} = 1 \text{ Joule/sec}$$

$$P = F \times v \text{ (linear)}$$

$$P = \tau \times \omega \text{ (rotational)}$$

Horsepower:

Metric 1Hp = 736watts

Electrical Hp = 746watts

Mechanical Hp = 745.7watts

$$1 \text{ ft-lbf} / \text{sec} = 192 \text{ in-lbf} / \text{sec} = 1.356 \text{ watts}$$

# Force

Force is the property of imparting acceleration to particles or objects  
(It is the push or pull on an object as a result of its interaction with another object)

## Newton's Second Law of Motion:

Force on an object produces an acceleration (change in velocity)

$$F(\text{Newtons}) = \text{mass}(\text{kg}) \times \text{acceleration}(\text{meters/second}^2) \quad 1\text{N} = \text{kg} \cdot \text{m} \cdot \text{s}^{-2}$$

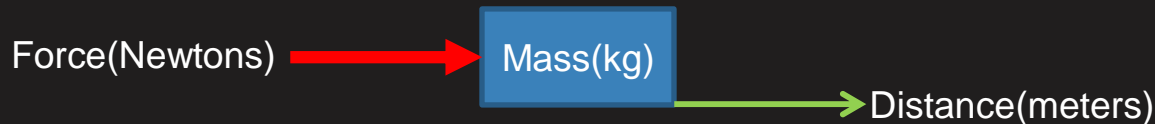
Kg is the standard for mass  
1kg = mass of 1 liter of water

$$\text{Acceleration} = (V_f - V_i) / (t_f - t_i)$$

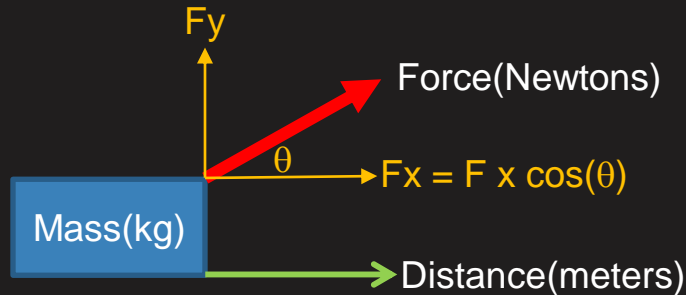
# Linear Force

Force is a vector; It has Magnitude and Direction

Linear Work:  $W = F \times d$



Linear Work:  $W = F \times \cos(\theta) \times d$



# Linear Force on Incline (Traction)

- $F_{\text{friction}} = \mu \times F_{\text{normal}}$  (Experimentally determine  $\mu$ )

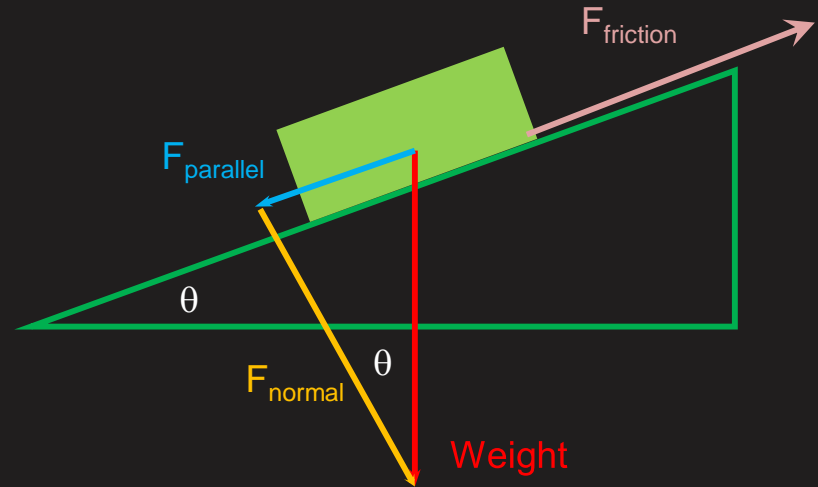
- $F_{\text{normal}} = \text{Weight} \times \cos(\theta)$

- $F_{\text{parallel}} = \text{Weight} \times \sin(\theta)$

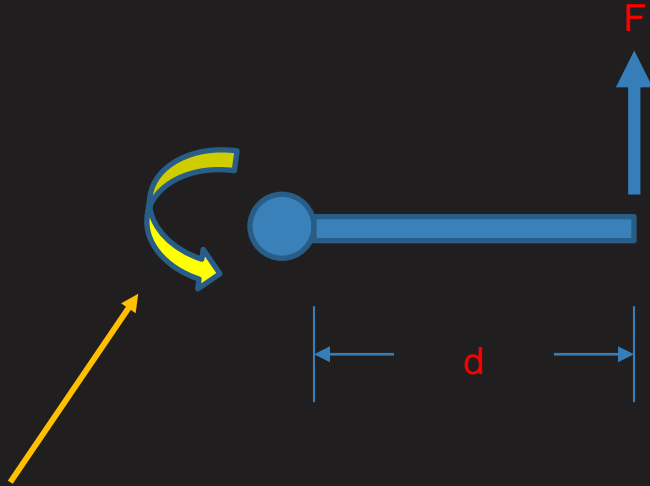
- When  $F_{\text{parallel}} = F_{\text{friction}}$ ; no slip

$$\text{Weight} \times \sin(\theta) = \mu \times \text{Weight} \times \cos(\theta)$$

$$\sin(\theta) / \cos(\theta) = \mu = \tan(\theta)$$



# Rotational Force



Torque =  $F$ (force perpendicular) x  $d$ ( distance of the lever arm)

$(2 * \text{Pi}) / 60 \text{ RPM} = 1 \text{ rad/ sec}$   
 $1 \text{ RPM} = 0.105 \text{ rad / sec}$   
 $1 \text{ MPH} = 0.45 \text{ m / sec}$

Kinetic energy =  $\frac{1}{2} \times I \times \omega^2$

Power =  $\tau \times \omega$  (Nm x rad / sec)

lbf-in = 0.112984829 N-m

lbf-ft = 1.36 N-m

# Weight(lb) is a Force

$g = \text{gravity } (9.8\text{m/s}^2)$

Mass(Slug)

Note:

1 slug = 14.5932 Kg

1 slug = 32.1740 lbm

1 kg = 2.2046 lbm

1 lbm = 0.4536 Kg

Since weight is a force its SI unit is N

1 N = 0.2248 lbf

1 lbf = 4.448 N

$$\text{Weight(lbs)} = m \times g$$

Standard  $g = 9.8\text{m/s}^2$  or  $32.2 \text{ft/s}^2$   
( $32.1740486 \text{ft/s}^2$  or  $9.80665 \text{m/s}^2$ )

*Under standard gravity: 1lbm weights 1lbf*

# FIRST Force Generation Options

## Pneumatic



Air Cylinder



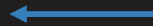
- 2 position action
- Slow movement  
(Time Required:  
pressurizing air lines)

## Electromagnetic



Solenoid

DC motor



- Variable position
- Slow movement  
(Time Required:  
create mag field,  
overcome inertia)

## Mechanical



Spring



- Push/pull action
- Fast movement  
(stored energy)

# Mechanical Advantage

Mechanical Advantage (MA) is a measure of force amplification achieved by using a tool  
Wikipedia

$$MA = \text{Force}_{\text{out}} / \text{Force}_{\text{in}}$$

However, Conservation of energy: Energy In = Energy Out + Losses

## Simple machine tools:

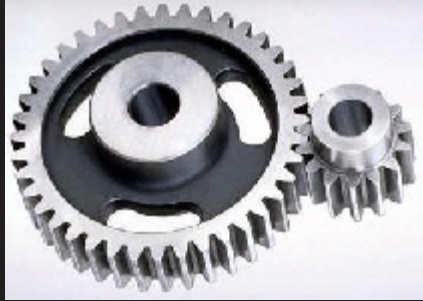
Lever	MA = length in / length out
Ramp	MA = ramp length / ramp height
Pulley	MA = effort distance / Load distance
Screw	MA = Rotation / Pitch
Wheel and axle	MA = Wheel radius / Axle radius

## Note:

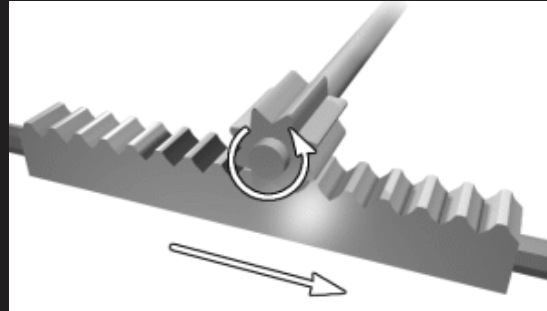
Wedge – double incline plane

Gear – series of levers

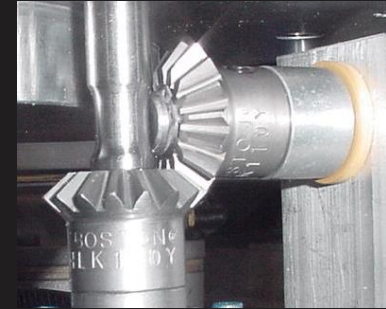
# MA-Gear Types



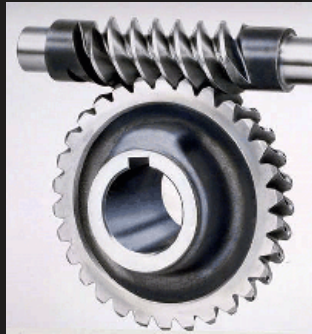
Spur



Rack and Pinon



Bevel



Worm

## Gear Types:

Spur – most common

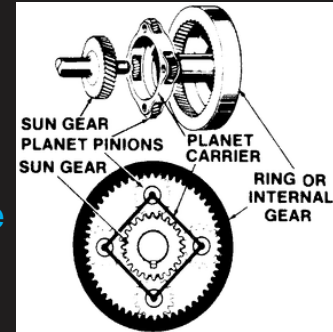
Helical - less noise than a spur

Rack & Pinion – rotary to linear

Worm gear – analogous to a screw – acts as a brake when stopped

Internal gear – used in planetary gear arrangement

Bevel gear – transmits motion at 90 degrees



Planetary

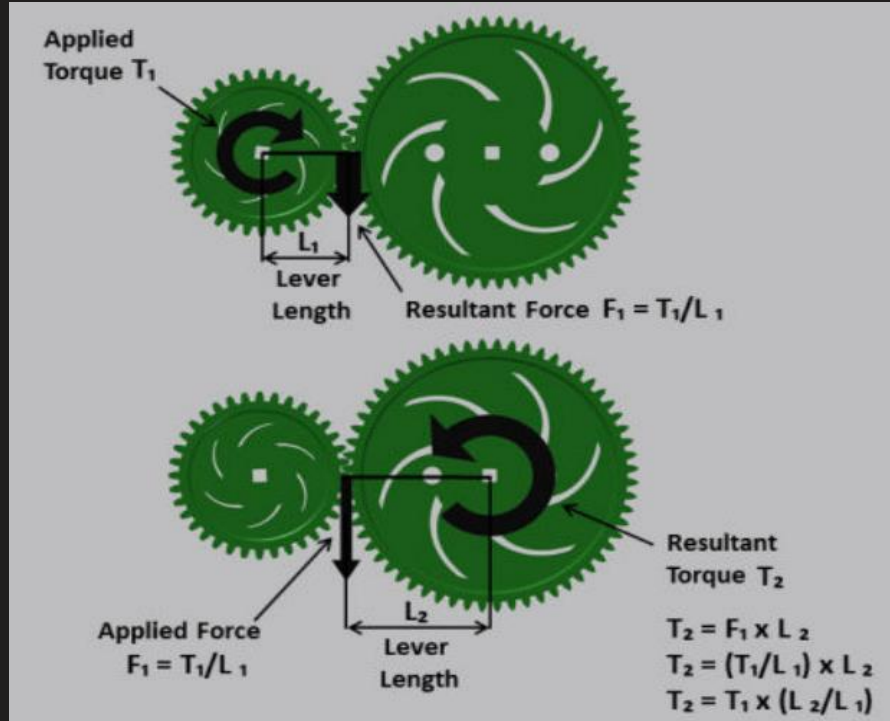
# MA – Why use gearing?

A motor is more efficient at high speeds in that it uses less current to deliver power. As we will see, gearing allows the user to swap torque and speed.

However:

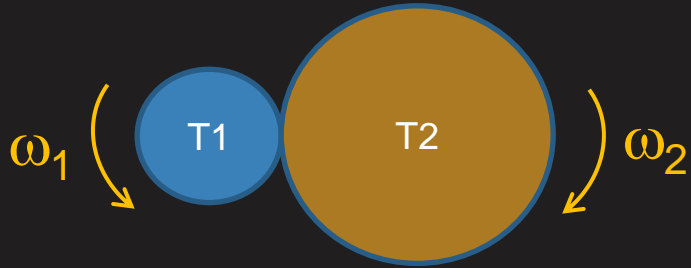
$$\text{energy in} = \text{energy out} + \text{losses}$$

# MA-Gearing is a series of levers



Vex Robotics

# Gears – Gear Ratio



$N > 1$ :

- Decreased Speed
- Increased Torque

$N < 1$ :

- Increased Speed
- Decreased Torque

Gear Ratio(N) = #Input turns / #Output turns

Gear Ratio(N) = radius2 / radius1

Gear Ratio(N) = #Teeth(T2) / #Teeth(T1)

Torque:  $\tau_{out} = \tau_{in} \times N$

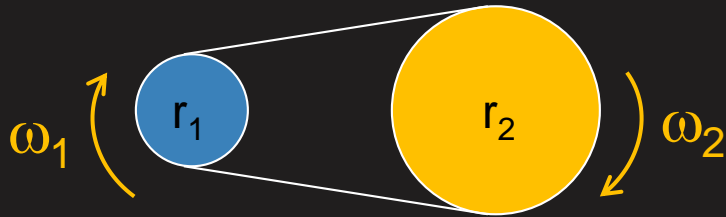
Speed:  $\omega_{out} = \omega_{in} \times (1 / N)$

Gear Efficiency( $\eta$ ) =  $P_{out} / P_{in}$

Torque:  $\tau_{out} = \tau_{in} \times N \times \eta$

Speed:  $\omega_{out} = \omega_{in} \times (1 / N) \times \eta$

# MA – Gear Ratio: Belt / Chain Drive

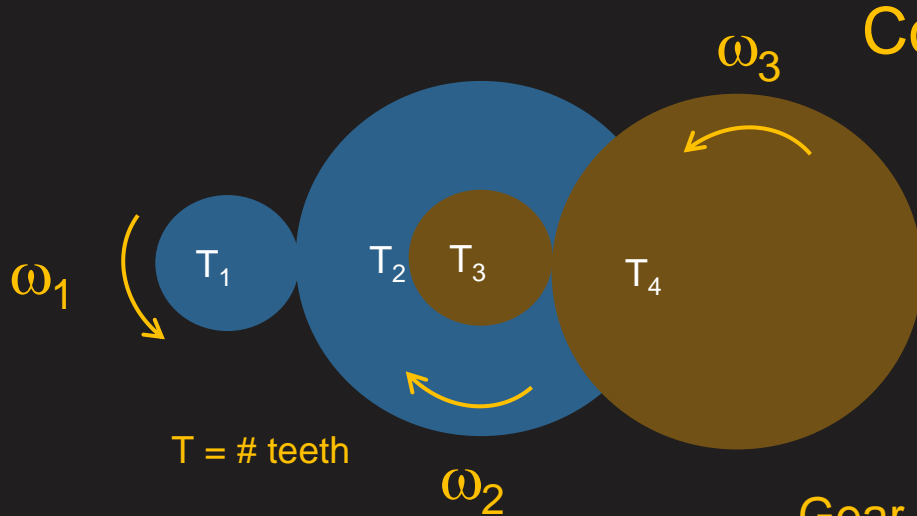


Gear Ratio(N) = Driven Pulley( $r_2$ ) / Driving Pulley( $r_1$ )

Torque:  $\tau_{out} = \tau_{in} \times N$

Speed:  $\omega_{out} = \omega_{in} \times (1 / N)$

# MA-Gear Train



Consecutive gear stages multiply:

$$\text{Gear Ratio: } N_t = N_1 \times N_2 = (T_2 / T_1) \times (T_4 / T_3)$$

$$\text{Torque: } \tau_{\text{out}} = \tau_{\text{in}} \times N_t$$

$$\text{Speed: } \omega_{\text{out}} = \omega_{\text{in}} \times (1 / N_t)$$

$$\text{Torque: } \tau_{\text{out}} = \tau_{\text{in}} \times N_t \times \eta_1 \times \eta_2$$

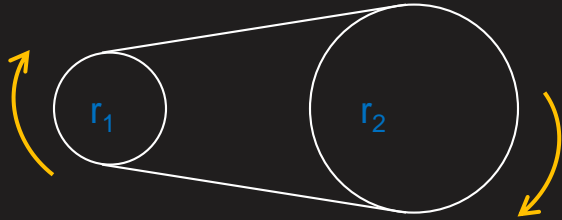
# MA-Gear Efficiency

Remember, wear and lubrication will also dramatically affect gear efficiencies

## 1. Chain & Belt

Efficiency ~ 85% - 98%

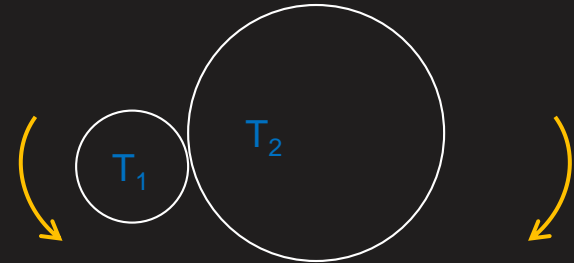
$$\text{GR } N = r_2/r_1$$



## 2. Spur Gears

Efficiency ~ 95% - 98%

$$\text{GR} = T_2/T_1$$

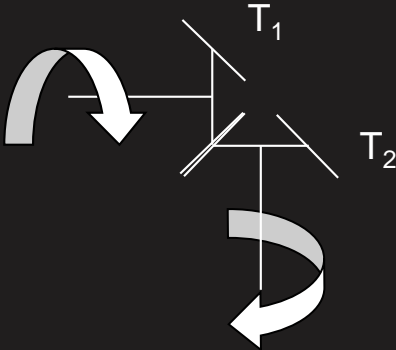


# MA-Gear: Bevel/Worm Gear Efficiency

### 3. Bevel Gears

Efficiency ~ 90% - 95%

$$GR N = T_2/T_1$$



### 4. Worm Gears

Efficiency ~ 40% - 70%

$$GR N = \frac{\text{\# Teeth on Worm Gear}}{\text{\# of Threads on worm}}$$

# MA-Gear: Planetary Gear- Efficiency

## 5. Planetary Gears

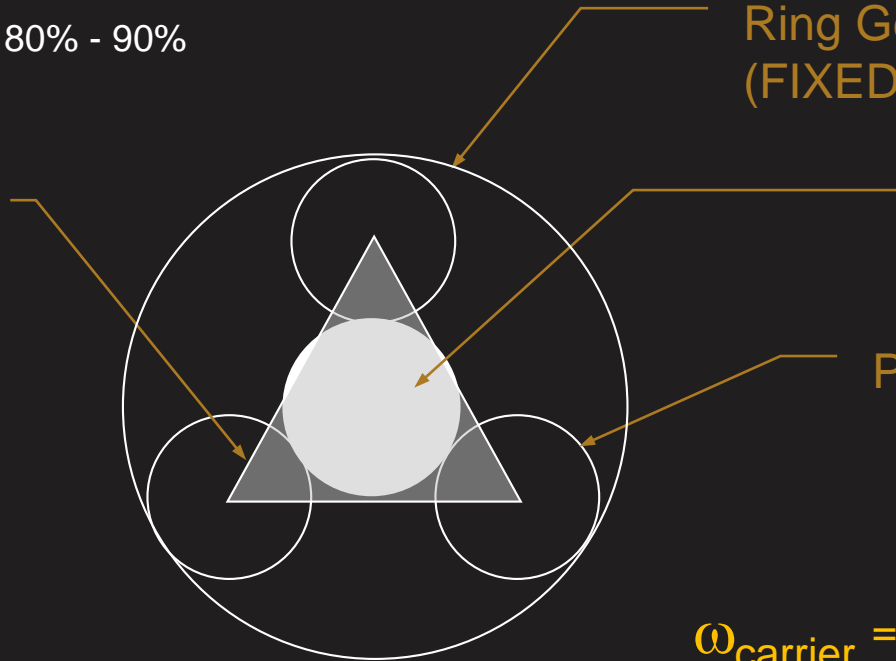
Efficiency ~ 80% - 90%

Carrier  
(OUTPUT)

Ring Gear  $T_2$   
(FIXED)

Sun Gear  $T_1$   
(INPUT-Driven)

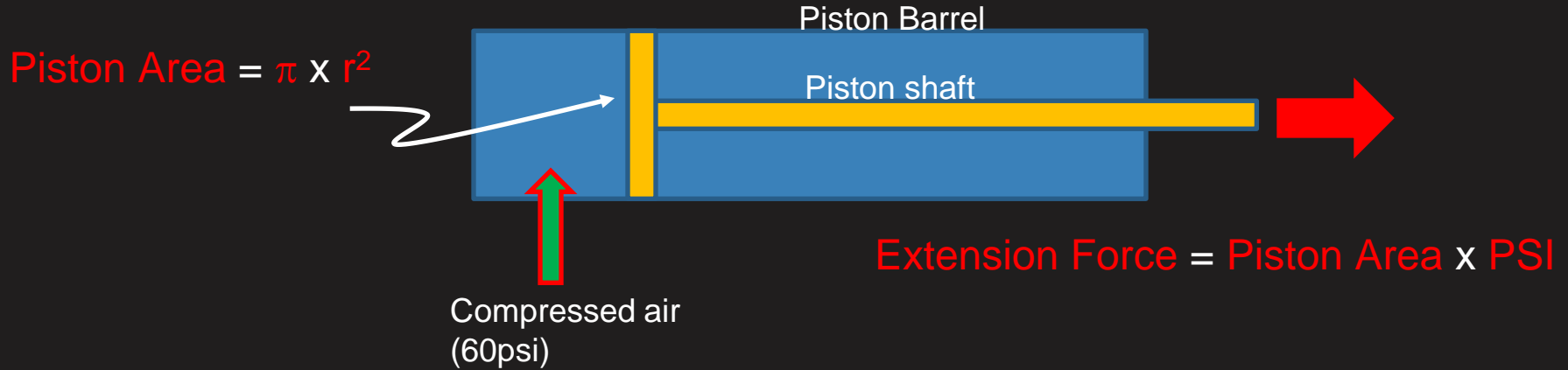
Planet Gear



$$GR = \frac{\#teeth\_ring}{\#teeth\_sun} + 1$$

$$\omega_{carrier} = (1 + (T_1 / T_2)) \times \omega_{sun}$$

# MA-Piston



Note:

$$\text{Retraction Force} = (\text{Piston Area} - \text{Piston Shaft Area}) \times \text{PSI}$$

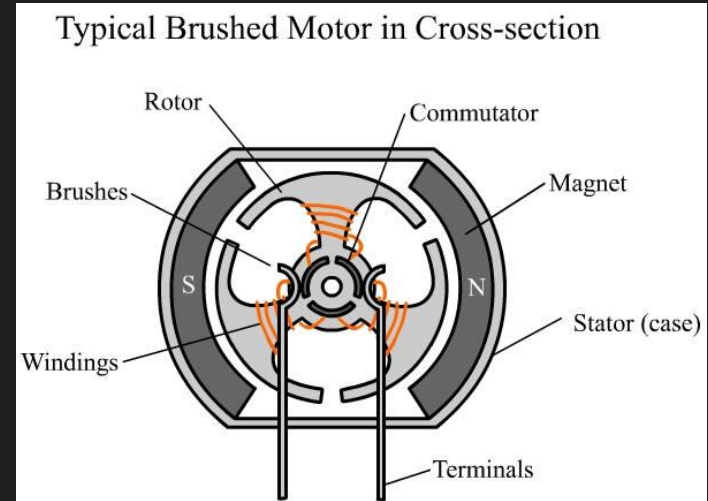
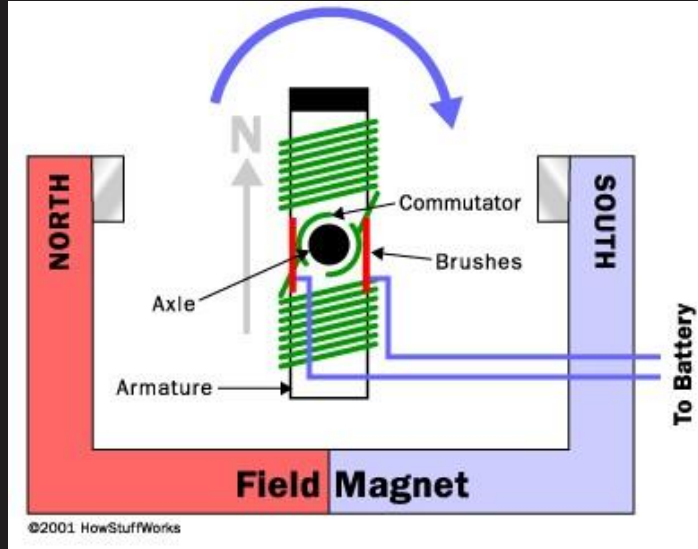
# Motors

Goals of this section:

- 1 Motor design and FIRST control system layout
- 2 FRC acceptable motors
- 3 DC motor specs and speed-torque curves
- 4 Electrical circuit resistance
- 5 Motor selection process

# Motor Design

A **motor** is a special electromagnet that changes **electrical** energy to rotary **mechanical** energy to produce **torque** that can do **work**



# FIRST Motor Control Components/Process



12Volt Battery

12 Volts



Power Distribution

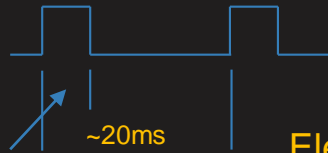
12 Volts



System Controller

Cmd: set(pwr level);  
(Pwr level= -1 to 1)  
1ms to 2ms (1.5ms=Zero)

PWM(PPM)



Electronic Speed Controller



Motor

# FRC Motors

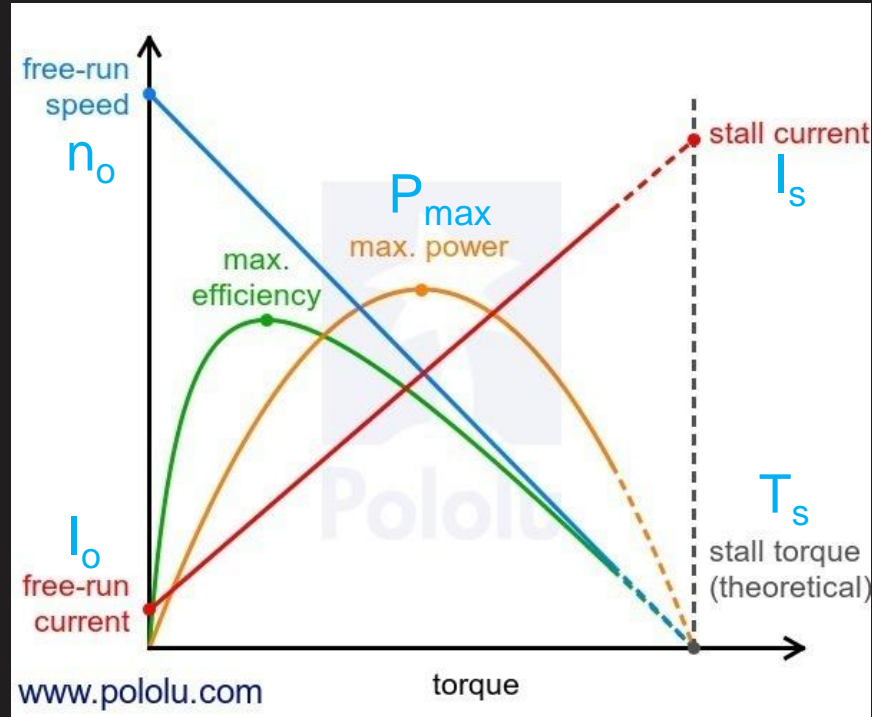
2015 FRC Motor Information

Make	Name	Part Number	Max Power (W)	Stall Torque (oz-in)	Free Speed (rpm)	Free Current (A)	Stall Current (A)	Weight (lbs)
AndyMark	9015 Motor	am-0912	179.49	60.64	16000	1.2	63.80	0.50
AndyMark	RS775-5 Motor	am-2161	36.91	35.00	5700	0.6	22.00	0.81
AndyMark	RS775-125 Motor	am-2194	36.91	35.00	5700	0.6	22.00	0.78
AndyMark	Snow Blower Motor	am-2235	29.60	1600.00	100	5.0	24.00	1.11
BaneBots	RS-550 Motor	M5-RS550-12 M5-RS550-12-B	245.83	68.85	19300	1.4	85.00	0.48
BaneBots	RS-775 Motor	M7-RS775-18	266.79	110.93	13000	1.8	86.70	0.74
Bosch	Spindle Drive Motor	6004 RA3 353-01	N/A	N/A	6500	2.5	21.00	1.10
Denso	Window Motor	262100-3030	23.33	1501.10	84	1.8	18.60	1.02
Denso	Window Motor	262100-3040	23.33	1501.10	84	1.8	21.00	1.02
Denso	Throttle Motor	AE235100-0160	18.04	18.40	5300	1.0	7.00	0.50
Varies	CIM Motor	Varies	337.34	343.40	5310	2.7	133.00	2.80
VEX	BAG Motor	217-3351	146.71	56.64	14000	1.8	41.00	0.71
VEX	Mini CIM Motor	217-3371	227.40	198.26	6200	1.5	86.00	2.16

# Motor Characteristics

## Motor Characteristics from Data Sheet

- Max Power
- Stall Torque
- Stall Current
- Free running Current
- Free running Speed

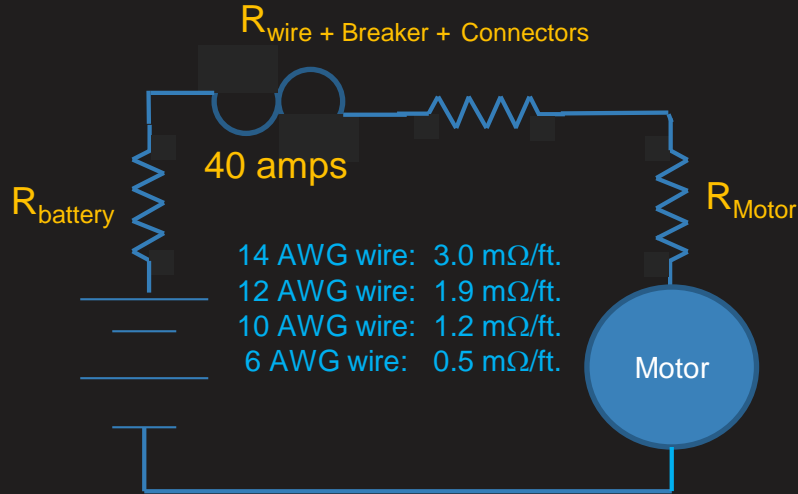


# CIM Data from Vex

	Published Specification	Experimental Data
Free Speed (RPM)	5310 (+/- 10%)	5330
Free Current (A)	2.7	2.7
Maximum Power (W)	337	337
Stall Torque (N · m)	2.42	2.41
Stall Current (A)	133	131



# Circuit resistance Matters



## System Resistance:

Battery	0.012
Wires(10ft of 12AWG)	0.019
Breakers, connectors	0.020
Total	0.051

Motor(nom) – CIM 0.090 ohms

$$R_m = V_{\text{spec}} / I_{\text{stall}} = 12 / 133 = 0.090$$

Motor(hot-increase 40% - CIM) 0.126

Total System 0.177

Additional resistance reduces stall torque proportionally

$$\text{By } R_{\text{motor}} / R_{\text{system}} = 0.126 / 0.177 = 0.712$$

$$\tau_{\text{stall}} = 343.4\text{oz-in} \times 0.712 = 244.5 \text{ oz-in}$$

# Available Motor Power

Facts:

- 1) Each motor fused for 40 amps
- 2) The batteries start out between 13-14v fully charged and end up around 11V when mostly discharged (empty)

A) Available electrical power.

$$11\text{v} * 40\text{A} = 440 \text{ Electrical Watts } \textit{per motor}$$

B) most motors are 40-60% efficient when converting electrical energy to mechanical energy. Maximum sustainable mechanical power available per motor:

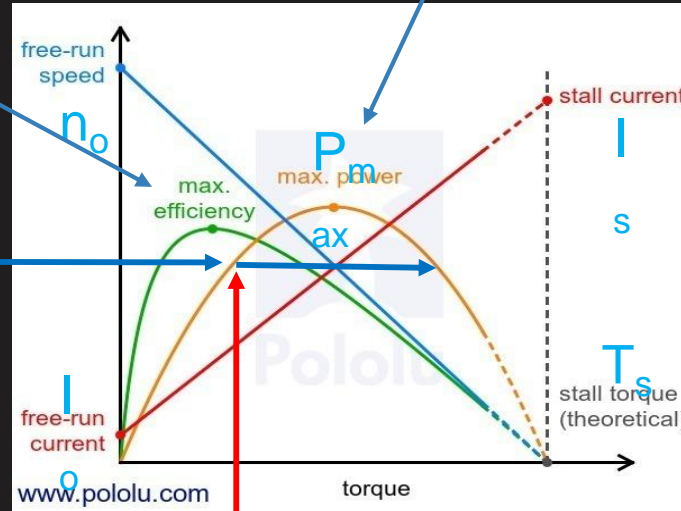
$$440\text{Watts per motor} * 0.4 = 176\text{Watts}$$

# Motor Selection

Max Efficiency: ~25% Stall Torque or ~60% Max Power

Max Power: 50% Stall Torque, ~ 50% Stall Current, and 50% Free-running speed

Power is available at 2 different operating conditions:  
1 High speed / low torque  
2 Low speed / high torque



Select this power point for design:

- 1) For lower current draw
- 2) Use of mechanical advantage to regain torque

# Best practices

- 1) Operate motor on left side of performance map (high speed / low torque)
- 2) Consider space and weight of motors in robot design
- 3) Air-cooled motors cannot operate near stall for more than a few seconds
- 4) Less gear trains better energy transfer
- 5) High gear: 14-16 ft/sec, low gear: 5-6 ft/sec
- 6) Reduce side load on motor output shaft
- 7) Use springs to balance motor energy

# Motor Selection Process

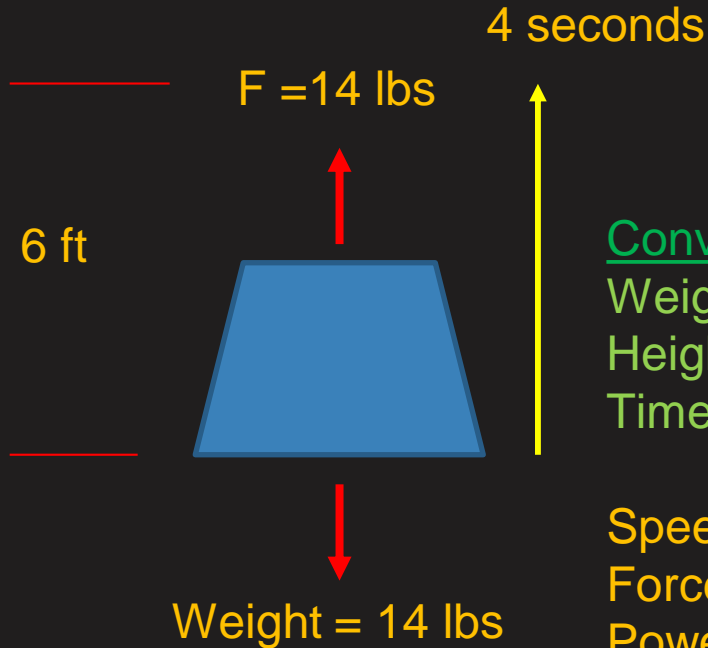
- 1) Define the mechanical system
- 2) Convert all parameters to SI units
- 3) Determine Power to accomplish task
- 4) Find motor for application (FIRST approved Motors)
- 5) Determine mechanical advantage needed
- 6) Order motor and gear box

# Work examples

Typical FIRST robotics work calculations:

- 1 Lifting a weight
- 2 Lifting a weight with a winch
- 3 Lifting a robot
- 4 Moving a robot
- 5 Lifting an object with an arm
- 6 Moving an object with a screw

# Work: Lift a weight



1 kg = 2.2 lbm | 1 lbm = 0.4536 kg  
1 m = 3.281 ft | 1 ft = 0.3048 m  
Standard gravity  $g = 9.8 \text{ m/s}^2$   
1 N = 0.2248 lbf | 1 lbf = 4.448 N  
1 Nm = 0.7376 ft-lbf | 1 ft-lbf = 1.3556 Nm  
1 ft-lbm / sec = 1.36 Watt  
1 rad/sec = 0.1047 RPM

## Convert to SI units:

Weight:  $W = 14 \text{ lb.} \approx 62 \text{ N}$  ( $F=ma$ )

Height:  $h = 6 \text{ ft.} \approx 1.8 \text{ m}$

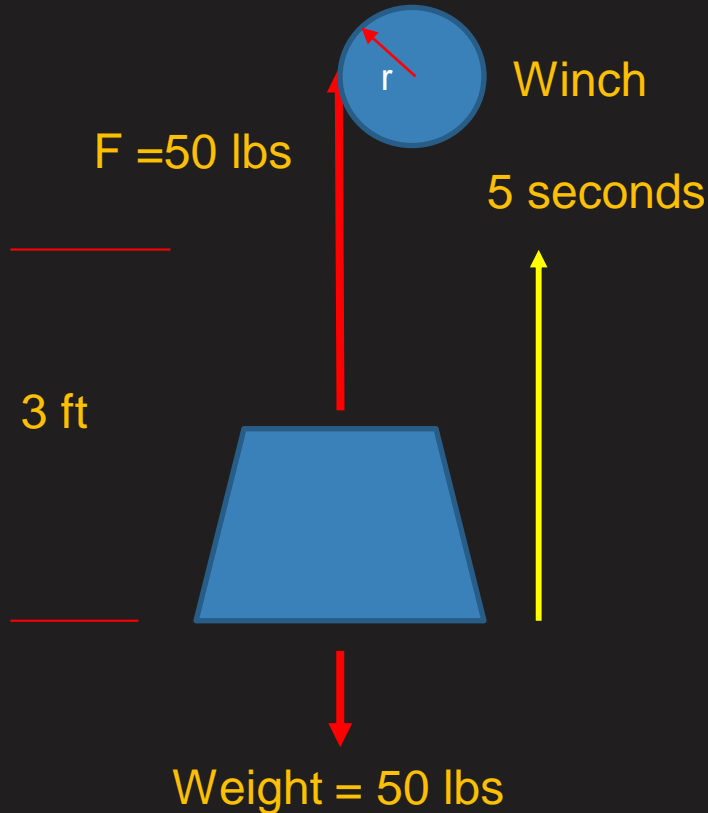
Time:  $t = 4 \text{ s}$

Speed:  $v = 1.8 \text{ m} / 4 \text{ s} = 0.45 \text{ m/s}$

Force:  $F = W = 62 \text{ N}$

Power:  $P = F \times v = 62 \text{ N} \times 0.45 \text{ m/s} = 28 \text{ Nm/sec} = 28 \text{ W}$

# Work/Power: Lift a weight (What is r)



1 kg = 2.2 lbm | 1 lbm = 0.4536 kg

1 m = 3.281 ft | 1 ft = 0.3048 m

Standard gravity  $g = 9.8 \text{ m/s}^2$

1 N = 0.2248 lbf | 1 lbf = 4.448 N

1 Nm = 0.7376 ft-lbf | 1 ft-lbf = 1.3556 Nm

1ft-lbm / sec = 1.36 Watt

1 rad/sec = 0.1047 RPM

$$\text{Power} = ((50 \text{ lb})(3 \text{ ft})/5 \text{ sec})(1.36\text{W} / 1 \text{ ft-lbm/sec}) = \sim 40\text{W}$$

Convert to SI units:

Weight:  $W = 50 \text{ lb.} = \sim 222 \text{ N}$  ( $F=ma$ )

Height:  $h = 3 \text{ ft.} = \sim 0.914 \text{ m}$

Time:  $t = 5 \text{ s}$

Speed:  $v = 0.914 \text{ m} / 5 \text{ s} = 0.183 \text{ m/s}$

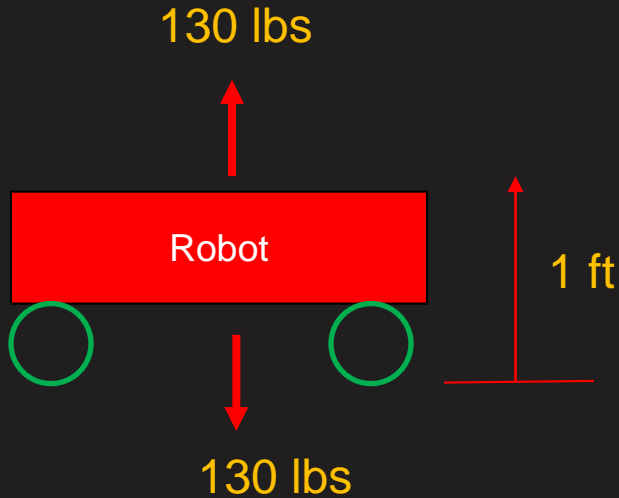
Force:  $F = W = 222 \text{ N}$

Power:  $P = F \times v = 222 \text{ N} \times 0.183 \text{ m/s} = \sim 40 \text{ Nm/sec} = 40 \text{ W}$

A motor selected at 40W (~45% eff) is 100 in-lbs;

Thus  $r = 100\text{in-lbs} / 50 \text{ lbs} = 2\text{in}$

# Work/Power: Lift robot



1 kg = 2.2 lbm | 1 lbm = 0.4536 kg  
1 m = 3.281 ft | 1 ft = 0.3048 m  
Standard gravity g = 9.8 m/s<sup>2</sup>  
1 N = 0.2248 lbf | 1 lbf = 4.448 N  
1 Nm = 0.7376 ft-lbf | 1 ft-lbf = 1.3556 Nm  
1ft-lbm / sec = 1.36 Watt  
1 rad/sec = 0.1047 RPM

## Convert to SI units:

$W = 130 \text{ lbs} = 580 \text{ N (F=ma)}$   
 $H = 1 \text{ ft} = 0.31 \text{ m}$

$P = (\text{F})\text{orce} \times (\text{v})\text{elocity}$

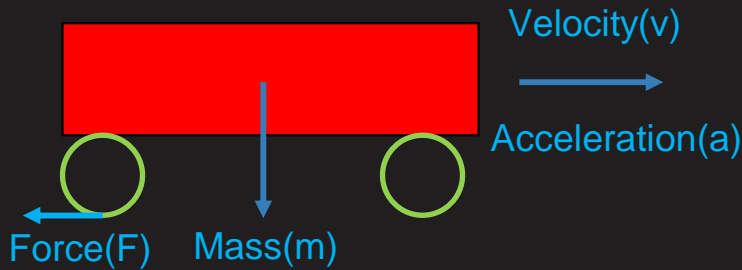
For 2 seconds:

$P = 580\text{N} \times (0.31\text{m} / 2\text{sec}) = 87 \text{ Watts}$

For 4 seconds:

$P = 580\text{N} \times (0.31\text{m} / 4\text{sec}) = 45 \text{ Watts}$

# Work/Power: Move a robot



Two wheels are driven

	CIM Ratio Aprox. Output Speeds (Loaded)	
4" Wheel	15.0 : 1 = 6 ft/s	5.13 : 1 = 16 ft/s
6" Wheel	16.4 : 1 = 8 ft/s	7.95 : 1 = 15.5 ft/s

Large wheels = faster, less torque  
Smaller wheels = slower, more torque

1 kg = 2.2 lbm | 1 lbm = 0.4536 kg

1 m = 3.281 ft | 1 ft = 0.3048 m

Standard gravity  $g = 9.8 \text{ m/s}^2$

1 N = 0.2248 lbf | 1 lbf = 4.448 N

1 Nm = 0.7376 ft-lbf | 1 ft-lbf = 1.3556 Nm

1ft-lbm / sec = 1.36 Watt

1 rad/sec = 0.1047 RPM

## Convert to SI units:

Mass:  $m = 150 \text{ lb.} = 68 \text{ kg}$

Speed:  $v = 6 \text{ ft./s} = 1.8 \text{ m/s}$

Acceleration:  $a = 1.8 \text{ m/s per sec} = 1.8 \text{ m/s}^2$

Force =  $m \times a = 68 \text{ kg} \times 1.8 \text{ m/s}^2 = 122 \text{ N}$

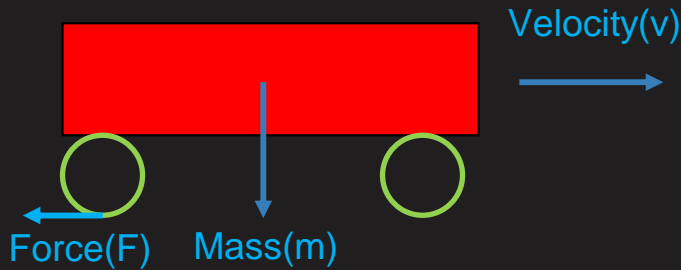
Force from each wheel:  $F = 122 \text{ N} / 2 = 61 \text{ N}$

Power:  $P = F \times v = 61 \text{ N} \times 1.8 \text{ m/s} = 110 \text{ W}$

# Work: Move a robot

Problem: (v)elocity = 1.8 m/s; (F)orce = 61 N

Motor selected:  $\omega_{\text{free}} = 559 \text{ rad/sec}$ ,  $\tau_{\text{stall}} = 1.2 \text{ Nm}$



Two wheels are driven

8inch wheels:  $R_{\text{wheel}} = 4'' = 0.1 \text{ m}$

1 kg = 2.2 lbm | 1 lbm = 0.4536 kg

1 m = 3.281 ft | 1 ft = 0.3048 m

Standard gravity  $g = 9.8 \text{ m/s}^2$

1 N = 0.2248 lbf | 1 lbf = 4.448 N

1 Nm = 0.7376 ft-lbf | 1 ft-lbf = 1.3556 Nm

1 ft-lbm / sec = 1.36 Watt

1 rad/sec = 0.1047 RPM | 1 RPM = 9.5493 rad/sec

Motor speed:  $\omega_{\text{motor}} = \omega_{\text{free}} / 2 = 559 \text{ rad/s} / 2 = 280 \text{ rad/s}$

Wheel speed:  $\omega_{\text{motor}} = v / R_{\text{wheel}} = (1.8 \text{ m/s}) / (0.1 \text{ m}) = 18 \text{ rad/s}$

Gear ratio:  $N_g = \omega_{\text{motor}} / \omega_{\text{wheel}} = (280 \text{ rad/s}) / (18 \text{ rad/s}) = 16$

Usual limit per stage is 5:1 - need two stages.

Gear efficiency:  $\eta_g = 0.9 \times 0.9 = 0.81$

Wheel torque:

$\tau_{\text{wheel}} = \eta_g \times N_g \times \tau_{\text{stall}} / 2 = 0.81 \times 16 \times 1.2 \text{ Nm} / 2 = 7.8 \text{ Nm}$

Force:  $F = \tau_{\text{wheel}} / R_{\text{wheel}} = (7.8 \text{ Nm}) / (0.1 \text{ m}) = 78 \text{ N (OK)}$

# Work: Lift weight with arm

1 kg = 2.2 lbm | 1 lbm = 0.4536 kg

1 m = 3.281 ft | 1 ft = 0.3048 m

Standard gravity g = 9.8 m/s<sup>2</sup>

1 N = 0.2248 lbf | 1 lbf = 4.448 N

1 Nm = 0.7376 ft-lbf | 1 ft-lbf = 1.3556 Nm

1ft-lbm / sec = 1.36 Watt

1 rad/sec = 0.1047 RPM

Work = Force x Distance

Work = F<sub>1</sub> x d<sub>1</sub> + F<sub>2</sub> x d<sub>2</sub>

Convert to SI Units:

10lbs = ~ 44N (F=ma)

8 lbs = ~ 36N

3ft = ~ 0.941m

6ft = ~ 1.8m

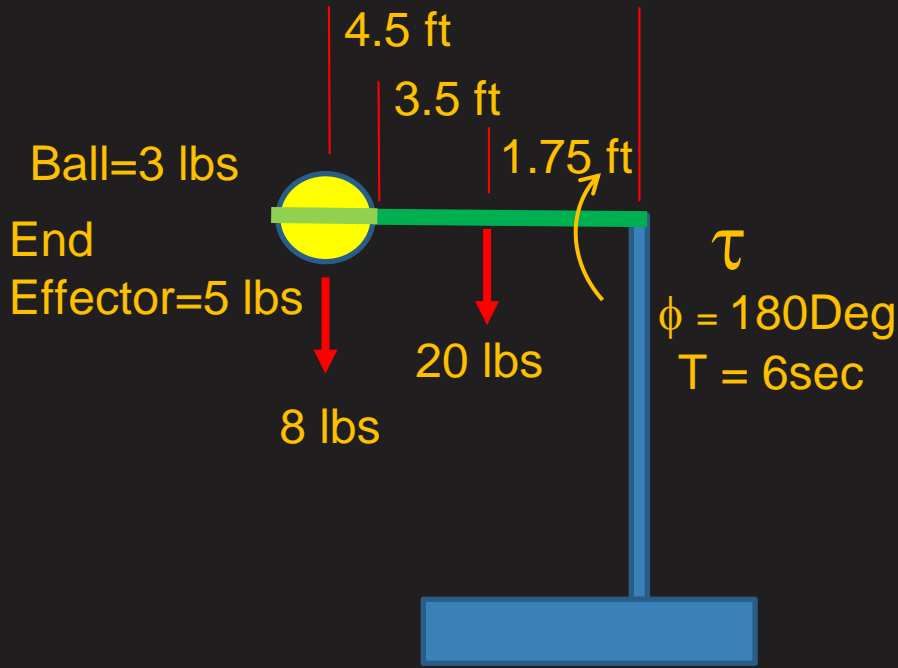
Work = 44N x 0.941m + 36N x 1.8m = ~ 106 Nm

$\tau$  = 106Nm x 1.356 = ~ 78 ft-lbs



(use center of gravity to determine distance)

# Work: Lifting a Ball (part1)



(use center of gravity to determine distance)

$$1 \text{ kg} = 2.2 \text{ lbm} \quad | \quad 1 \text{ lbm} = 0.4536 \text{ kg}$$

$$1 \text{ m} = 3.281 \text{ ft} \quad | \quad 1 \text{ ft} = 0.3048 \text{ m}$$

$$\text{Standard gravity } g = 9.8 \text{ m/s}^2$$

$$1 \text{ N} = 0.2248 \text{ lbf} \quad | \quad 1 \text{ lbf} = 4.448 \text{ N}$$

$$1 \text{ Nm} = 0.7376 \text{ ft-lbf} \quad | \quad 1 \text{ ft-lbf} = 1.3556 \text{ Nm}$$

$$1 \text{ ft-lbm} / \text{sec} = 1.36 \text{ Watt}$$

$$1 \text{ rad/sec} = 0.1047 \text{ RPM} \quad | \quad 1 \text{ RPM} = 9.5493 \text{ rad/sec}$$

$$\begin{aligned} \text{Total torque} &= \text{torque of ball} + \text{torque of arm} \\ &= (8 \text{ lbs} \times 4.5 \text{ ft} + 20 \text{ lbs} \times 1.75 \text{ ft}) \\ &= 71 \text{ ft-lbs} = 852 \text{ in-lbs} \approx 96 \text{ Nm} \end{aligned}$$

$$\begin{aligned} \text{Angular Velocity} &= 180 \text{ degrees} / 6 \text{ sec.} \\ &= 5 \text{ RPM} = 0.5 \text{ Rad/sec} \end{aligned}$$

$$\text{Power} = \text{Torque} \times \text{Angular Velocity}$$

$$\text{Power} = 96 \text{ N-m} \times 0.5 \text{ Rad/sec} = 48 \text{ Watt}$$

# Work: Lifting a Ball (part2)

Best to design a gear ratio such that the load reflected back to motor is around 20%~50% stall torque when motors are most happy.

CIM Motor selected: stall torque:  $2.42\text{Nm} = 1.785\text{ ft-lbs} \approx 21\text{ in-lbs}$

20%~50% stall torque =  $4.2\text{ in-lb} \sim 10\text{ in-lb}$

Torque range =  $852\text{ in-lb} / 4.2\text{ in-lb} \sim 852\text{ in-lb} / 10\text{ in-lb}$

Gear ratio required torque range =  $203:1 \sim 85:1$

We will run motor at 45% of stall torque with a little room before maximum motor power:

Working Torque =  $21\text{ in-lb} * .45 \approx 9\text{ in-lb}$

Gear ratio =  $852\text{ in-lb} / 9\text{ in-lb} = 95:1$

# Work: Lifting a Ball (part3)

Gear box option for CIM motor 100:1 two stage planetary gear.  
Efficiency = 85%

Effective gear ratio =  $10 \times 0.85 \times 10 \times 0.85 = 72:1$

Effective power = motor power \* total component efficiency...  
(We will just consider the gears' efficiency for this purpose)

.CIM 337W :

Effective power =  $337W \times .85 \times .85 = 243.5W$

Note: Marginal system – Improvements

- 1) Add torsion spring or surgical tubing to assist raising ball
- 2) Add chain drive with gear ratio > 1
- 3) Add another CIM/gearbox to other end of shaft (torque adds)

# Work: Screw example

Problem: screw (v)elocity = 0.45 m/s and need a (F)orce = 61 N

Selected motor:  $\omega_{\text{free}} = 2513 \text{ rad/sec}$ ,  $\tau_{\text{stall}} = 0.28 \text{ Nm}$

Screw speed = motor speed:  $\omega_{\text{screw}} = \omega_{\text{free}} / 2 = 2513 \text{ rad/s} / 2 = 1256 \text{ rad/s}$

$$\omega_{\text{screw}} = (1256 \text{ rad/s}) / (2\pi \text{ rad/revolution}) = 200 \text{ rev./s}$$

$\tau_{\text{screw}} = \tau_{\text{motor}} = \tau_{\text{stall}} / 2 = 0.28 \text{ Nm} / 2 = 0.14 \text{ Nm}$

Screw pitch:  $p = v / \omega_{\text{screw}} = (0.45 \text{ m/s}) / (200 \text{ rev./s}) = 0.00225 \text{ m/rev.} = 0.00036 \text{ m/rad}$   
 $p = 11 \text{ threads per inch}$

Force: (Assume screw efficiency = 20%)

$$F = \eta g \times \tau_{\text{screw}} / p = (0.2 \times 0.14 \text{ Nm}) / (0.00036 \text{ m/rad}) = 78 \text{ N (OK)}$$